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Comparative Study of Exponential Signal by an Interpolated DFT Algorithm

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Abstract— **In this paper, exponential signals in the frequency domain are accurately analyzed by an algorithm, and the peaks of the discrete Fourier transform (DFT) result are adopted to obtain parameters that include amplitudes, frequencies, damping, and phases. There are two steps for this algorithm: interpolated DFT and leakage elimination. Interpolated DFT refers to the three neighboring spectral lines at the peak of each mode used to calculate an approximate result, with the purpose of leakage elimination being to eliminate the influence of leakage on these data. After some iteration, this algorithm will obtain accurate parameters using Interpolated DFT algorithm. Also comparison with different algorithms and Theoretical values of the parameters are also obtained here.**

***Keywords*-dft,idft,damping,leakage**

I. Introduction

In physical systems, dynamic behavior can be expressed in differential equations. The results of any linear and time-invariant or differential equations are mostly composed of exponential forms.

In this paper, exponential signals in the frequency domain or accurately analyzed by an algorithm & the peaks of the discrete fourier transform(DFT) results are adopted to obtain parameters that include amplitude, frequencies, damping and phases. Thus, this two types of exponential forms, if the damping is equal to zero, the mode is periodic & if the damping is not equal to zero the mode is aperiodic & thus it will decay to zero with time.

If signal is periodic, the analysis method for periodic signal can be divided into time & frequency domain methods. In time domain, the quasi-newton method minimizes the distance between estimated signal and measured signal.

If signal is aperiodic, it can also be divided into time & frequency domain method.

Frequency domain can display the important messages that are hidden behind the signal, therefore the uses of such method can deal with computer system & quickly deduce solutions. This paper establishes an algorithm to analyze the parameter of exponential signals in the frequency domain.

In frequency domain, the mode energy will centralize near its frequency, therefore the frequency of a node will be located near its spectral peak. This paper considers that a spectral peaks on a spectrum is then number of modes. This peaks hides four unknown parameters, frequency, damping, amplitude & phase.

In this paper simple and effective formulas are given to analyze these parameters by reconstructing the spectrum of a signal, so that the parameters may be found accurately.

Consequently, there are four parts of the section:-
1) Calculate the parameters using DFT algorithm.
2) Calculate parameters using interpolated DFT algorithm.
3) The spectrum of an exponential signal.
4) Leakage elimination and influence of noise

In general, any signal consists of several independent modes. Every mode can be taken as exponential forms creating its own spectral peak on the spectrum.

II. Spectrum of Exponential Signal

When discrete signal x(n) is transformed into the frequency domain, it can be expressed as

$$x(n) = \sum_{k=1}^{K} A_k * e^{-\alpha_k n/N} \cos(2\pi f_k n / N + \phi_k)$$
$$n = 0,1,........, N-1$$

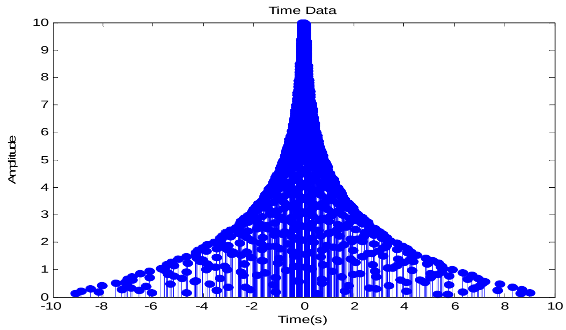
The exponential forms are taken as modes in this paper, consisting of k independent modes, the signal can be expressed as

$$x(n) = \sum_{k=0}^{K} x(n) * e^{-j2\pi mn/N}$$
$$m = 0,1,........., N-1$$

where,
A_K is the amplitude, ϕ_K is the phase,
α_K is the damping, f_K is the frequency
Where T is the whole measured sampling time.

Let us consider the input signal for analysis of frequency, damping, phase and amplitude is given as,

$i(t)=10e-0.5t \cos(2\pi\cdot 30.2t-1.047)+15e-10t\cos(2\pi\cdot 60.8t)$

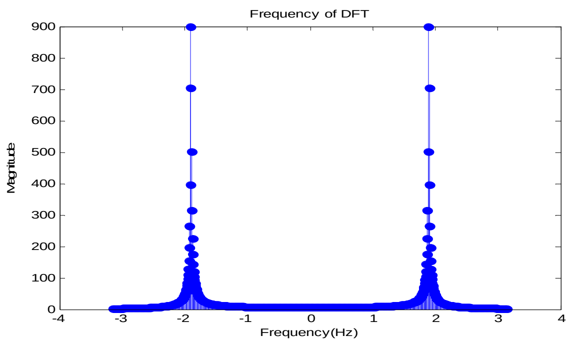


In frequency domain, every mode can be separated into two parts. One is located at the positive frequencies the another is located in the negative frequency.

A. Calculation of Frequency

$$X(m)=\sum_{k=1}^{K}(A_k^{+}(m)\angle\phi_k^{+}(m)+(A_k^{-}(m)\angle\phi_k^{-}(m))$$

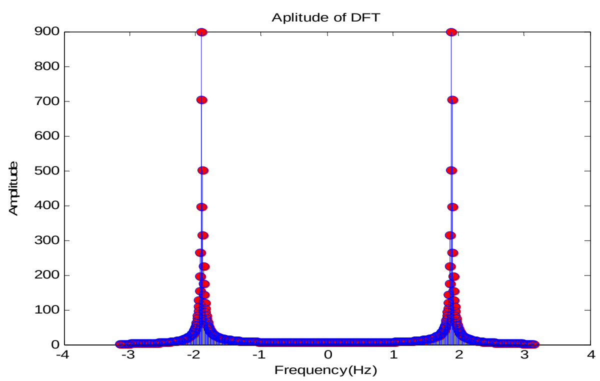
$$m=0,1,-----,N-1$$



B. Calculation of Amplitude

$$A_k^{+}(m)=A_k e^{\alpha_k(1-N)/(2N)}/2$$
$$*(\cosh\alpha_k-\cos 2\pi(f_k-m)/\cosh\alpha_k/N-\cos 2\pi(f_k-m)$$

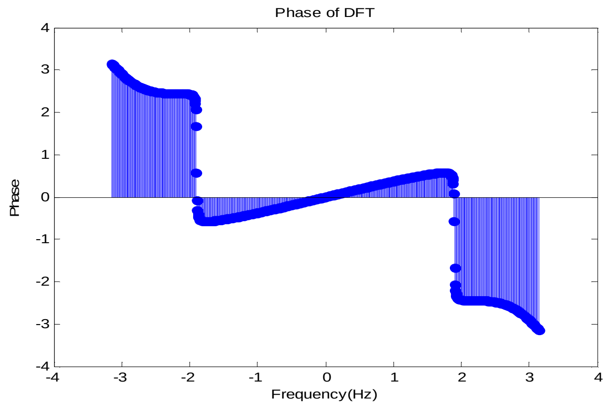
$$A_k^{-}(m)=A_k e^{\alpha_k(1-N)/(2N)}/2$$
$$*(\cosh\alpha_k-\cos 2\pi(f_k+m)/\cosh\alpha_k/N-\cos 2\pi(f_k+m)/N)^{1/2}$$



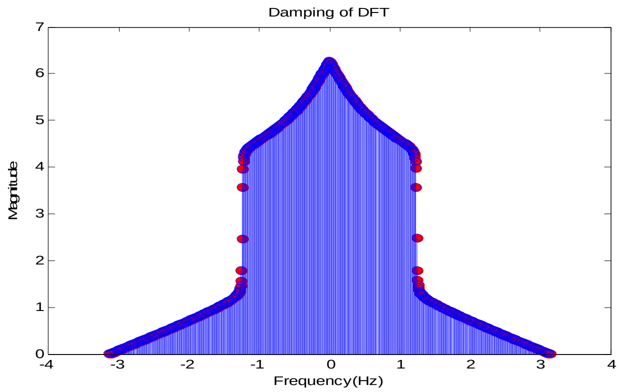
C. Calculation of Phase

$$\phi_k^{+}(m)=\phi_k-\tan^{-1}((\sin 2\pi(f_k-m)/e^{\alpha_k}-\cos 2\pi(f_k-m))$$
$$+\tan^{-1}(((\sin 2\pi(f_k-m)/N)/(e^{\alpha_k}-\cos 2\pi(f_k-m)/N))$$

$$\phi_k^{-}(m)=-\phi_k+\tan^{-1}((\sin 2\pi(f_k+m)/e^{\alpha_k}-\cos 2\pi(f_k+m))$$
$$-\tan^{-1}(((\sin 2\pi(f_k+m)/N)/(e^{\alpha_k}-\cos 2\pi(f_k+m)/N))$$



D. Calculation for damping



The first item of right-hand side is the influence of modes in the positive frequency & the second one is the influence of mode in the negative frequency. In positive frequency the amplitude & phase spectra are represented by similarly in negative frequency the amplitude & phase spectra is represented but in general we take the part in the positive frequency to represent the full spectrum. When the positive part is under calculation the negative part will become the interference. The interference from the negative part will be discussed with other interference.

III. INTERPOLATED DFT

For Interpolated DFT we consider the three spectral lines in the signal and the index of the peak is denoted by the letter p & the three highest amplitudes in a mode are denoted by

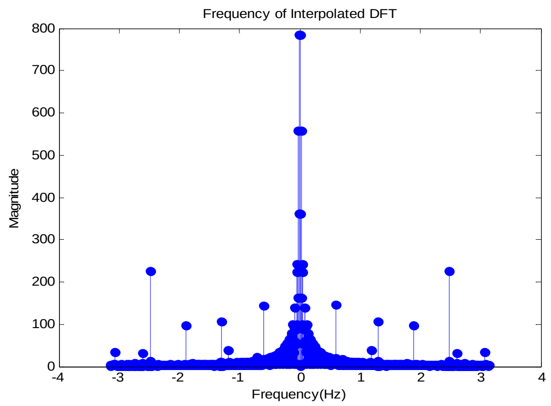
$$A(p)=A_p$$
$$A(p-1)=A_{p-1}$$
$$A(p+1)=A_{p+1}$$
$$\phi(p)=\phi_p$$

A condition must be satisfied that a mode has to include three spectral lines at least. This condition makes the frequency difference between any two modes larger than three frequency intervals. Application when the frequency difference between two modes is larger than light frequency intervals, the set of this formula could obtain satisfactory results.

1. Calculation of Frequency

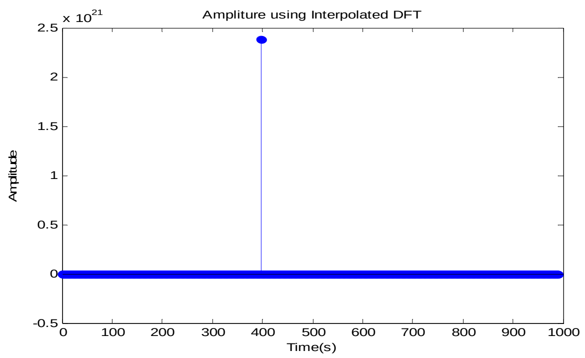
$$f = p + 0.5*((A_{p+1}^{-2} - A_{p-1}^{-2})/(2A_p^{-2} - A_{p+1}^{-2} - A_{p-1}^{-2}))$$

$$\delta = f - p$$



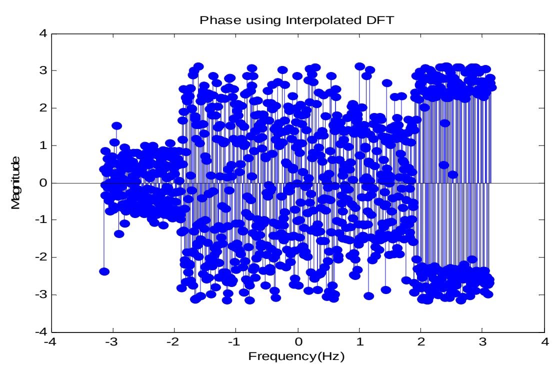
2. Calculation of Amplitude

$$A = \sqrt{2}/N * A_p * e^{\alpha/2} * ((\alpha^2 + (2\pi\delta)^2)/(\cosh\alpha - \cos 2\pi\delta))^{1/2}$$



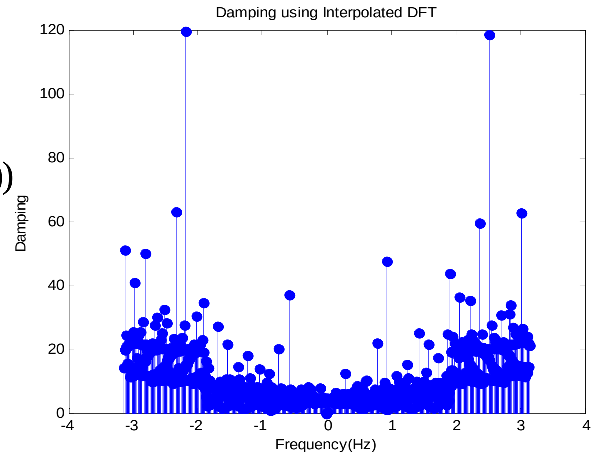
3. Calculation of Phase

$$\phi = \phi_p + \tan^{-1}(\sin 2\pi\delta / e^{\alpha} - \cos 2\pi\delta) - \tan^{-1}(2\pi\delta/\alpha)$$



4. Calculation for damping

$$\alpha = 2\pi*((A_{p+1}^{2}(\delta-1)^2 - A_{p-1}^{2}(\delta+1)^2)/(A_{p-1}^{2} - A_{p+1}^{2}))^{1/2}$$



Mode	Algorithm	Frequency	Damping	Amplitude	Phase
Mode-I	Real	30.2	0.5	10	-1.04
	DFT	30.83	0.4783	11.4132	-1.23
	Interpolated DFT	30.7872	0.5296	11.4132	-1.14
Mode-II	Real	60.8	10	15	0.785
	DFT	61.5458	10.359	15.4558	0.781
	Interpolated DFT	60.462	10.341	14.7441	0.785

Table(1): - Comparison between DFT and IDFT

IV. PROCEDURE

There are three major stages in this paper
1) Finding the parameter using different algorithms
2) Comparison between different algorithms
3) Leakage elimination

Step 1) Signal sampling:- To decide the whole measurement sampling time T, and the sampling rate R, the sampling time must be suitable which makes every mode easily recognizable and present excessive decay of exponential modes.

Step 2) Time frequency transformation: - The signal is transformed into a spectrum by the DFT

Step 3) Exponential modes will cause relative spectral peaks on the spectrums. A mode will decide the spectral peak on the spectrum. A mode can be decided by a spectral peak. The number of modes is the number of spectral peaks k. From every spectral peak, the reference data for p-1,p,p+1,Ap-1;Ap,Ap+1 & p can be obtained

Step 4) DFT: - The initial value of the frequency & then the initial value of damping, amplitude, phase also be calculated

Step 5) IDFT: - Finding the parameters using IDFT

Step 6) Compare the DFT & IDFT algorithms & their parameters for the same signal. We found some leakage is occurring in the signal.

Step 7) To eliminate the leakage we goes through leakage elimination process.

Step 8) According to found the parameters, the influence of the spectral peak can be obtained. Every mode calculation refers to three spectral line of its own peak.

Step 9) The leakage of spectral lines can be eliminated

Step 10) Draw the comparison table using other different algorithms

Step 11) Draw the leakage elimination table in which the first iteration only 20% of leakage is eliminate in second iteration 50% of leakage is eliminated & in 3rd iteration 100% leakage is eliminate.

V. CONCLUSION

Comparative study of exponential signal by an different algorithm has done. The result obtained is indicating that the Interpolated DFT is giving more accurate results.

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